Counting Principle & Permutations & Combinations

A license plate consists of 3 letters, followed by 3 digits.

How many different license plates are possible?

$\underline{26} \times \underline{26} \times \underline{26} \times \underline{10} \times \underline{10} \times \underline{10} = 17{,}576{,}000$

Suppose letters and digits may not be repeated. How many are possible?

$\underline{26} \times \underline{25} \times \underline{24} \times \underline{10} \times \underline{9} \times \underline{8} = 11{,}232{,}000$

Permutations

In how many ways can these letters be arranged? A B C

A ndy B eth C indy

$\underline{3} \times \underline{2} \times \underline{1}$

$= 6$ ways

ABC
ACB
CAB
CBA
BAC
BCA

Permutations

In how many ways can these letters be arranged? A B C D E F G

$\underline{7} \times \underline{6} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 5040$

"7 factorial" $= 7!$

How many ways could I choose just two letters from the list?

$\underline{7} \times \underline{6} = 42 \text{ ways} = {}_7P_2 = \frac{7!}{(7-2)!}$

Number of Permutations = The number of ways to select r objects from a group of n objects is

$${}_nP_r = \frac{n!}{(n-r)!}$$

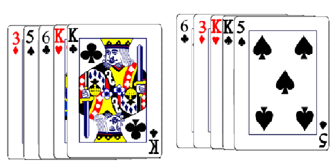
$\frac{7\cdot6\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}$

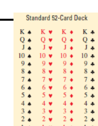
Find the number of permutations.

1. ${}_5P_3 = \frac{5!}{(5-3)!} = \frac{5\cdot4\cdot3\cdot\cancel{2}\cdot\cancel{1}}{\cancel{2}\cdot\cancel{1}} = 60$

2. ${}_4P_1 = \frac{4!}{(4-1)!} = \frac{4!}{3!} = \frac{4\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{\cancel{3}\cdot\cancel{2}\cdot\cancel{1}} = 4$

3. ${}_8P_5 = 6720$





A combination is a selection of "r" objects from a group of "n" objects where order is not important.

In a combination, ABC is the same as all of these:

ABC
ACB
CAB
CBA
BAC
BCA

Number of Combinations of r objects taken from a group of n objects is:

$${}_nC_r = \frac{n!}{(n-r)! \cdot r!}$$

${}_nC_r = \frac{52!}{(52-5)!\ 5!}$ Number of possible 5-card hands dealt from a 52-card deck.

Find the number of combinations.

1. ${}_5C_3 = \frac{5!}{2!\,3!} = \frac{5\cdot4\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{2\cdot1\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}} = 10$

2. ${}_4C_1 = 4$

3. ${}_8C_5 = 56$